ENGO 697

Remote Sensing Systems and Advanced Analytics

Session 5: How to develop radiative transfer models in SAR systems

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Outline

➔ Microwave RTMs

➔ The Snow Microwave Radiative Transfer (SMRT)

➔ Questions

Radiative Transfer Model

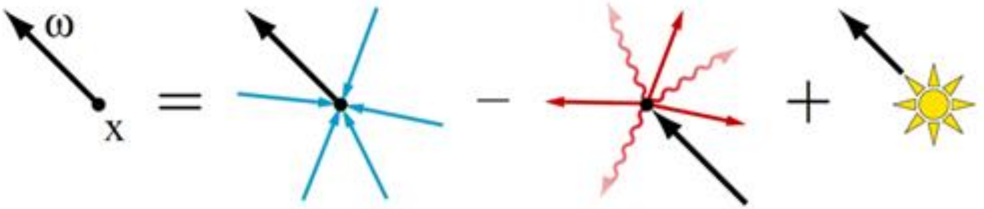
- **Radiation transfer** refers to the physical process of electromagnetic radiation transferring through a medium, which involves absorption, transmission, emission, and scattering processes.

- **Radiative Transfer Models** (RTMs) calculate the energy reflected, absorbed, emitted or transmitted as a function of other influencing factors in a plant canopy or planetary atmosphere.

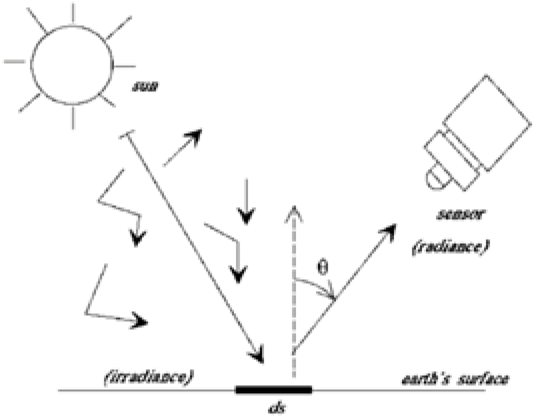
- RTMs can be used to predict the spectral transmission of the atmosphere, the light reflected or emitted from a plant, and the amount of energy absorbed or emitted at different levels.

Radiative Transfer Equations - how to describe ***<u>the variation of the radiance L per unit distance along ω?</u>***

The equation of radiative transfer simply says that ***as a beam of radiation travels***, it loses energy to absorption, gains energy by emission processes, and redistributes energy by scattering.



Differential radiance | In-scattering | Out-scattering & absorption | Emission



$$(\boldsymbol{\omega}\cdot\nabla)L(\boldsymbol{x},\boldsymbol{\omega}) = \sigma_s(\boldsymbol{x})\int_{\mathbb{S}^2} f_p(\boldsymbol{x},\boldsymbol{\omega}_i\to\boldsymbol{\omega})L(\boldsymbol{x},\boldsymbol{\omega}_i)\,\mathrm{d}\boldsymbol{\omega}_i - \sigma_t(\boldsymbol{x})L(\boldsymbol{x},\boldsymbol{\omega}) + Q(\boldsymbol{x},\boldsymbol{\omega})$$

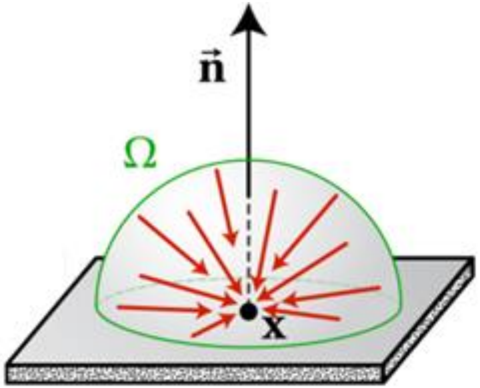
In-scattering | Out-scattering & absorption | Emission

Scattering coefficient: $\sigma_s(\boldsymbol{x}) \in \mathbb{R}_{\geq 0}$,
Phase function: $f_p(\boldsymbol{x},\boldsymbol{\omega}_i \to \boldsymbol{\omega})$, a probability density over $\mathbb{S}^2$ given $\boldsymbol{x}$ and $\boldsymbol{\omega}_i$

The ratio between σ_s and σ_t controls the fraction of radiant energy *not* being absorbed at each scattering and is also known as the *single-scattering albedo*

Extinction coefficient: $\sigma_t(\boldsymbol{x}) \in \mathbb{R}_{\geq \sigma_s(\boldsymbol{x})}$
σ_t controls how frequently light scatters and is also known as the *optical density*

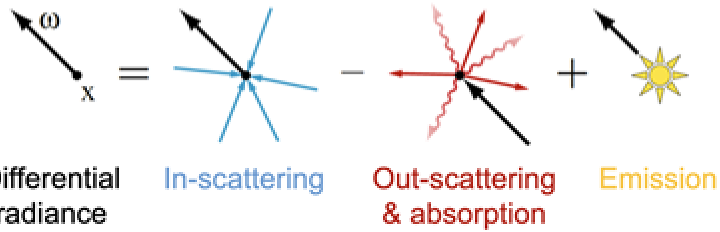
Source term: $Q(\boldsymbol{x},\boldsymbol{\omega}) \in \mathbb{R}_{\geq 0}$



- Differential radiance

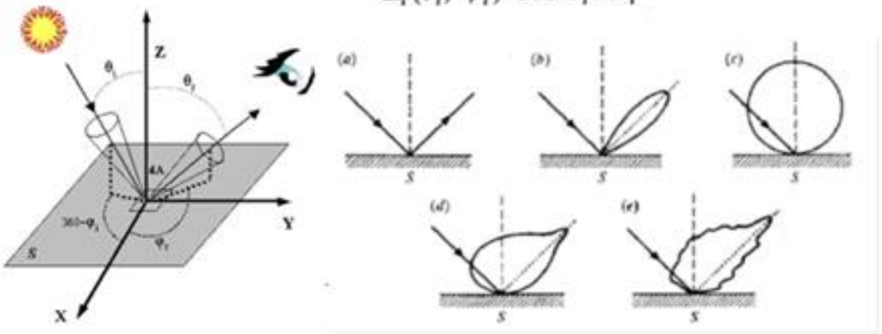
$$(\boldsymbol{\omega}\cdot\nabla)L(\boldsymbol{x},\boldsymbol{\omega}) = \left.\frac{\mathrm{d}L(\boldsymbol{x}+\tau\boldsymbol{\omega},\boldsymbol{\omega})}{\mathrm{d}\tau}\right|_{\tau=0} = \lim_{\tau\to 0}\frac{L(\boldsymbol{x}+\tau\boldsymbol{\omega},\boldsymbol{\omega}) - L(\boldsymbol{x},\boldsymbol{\omega})}{\tau}$$

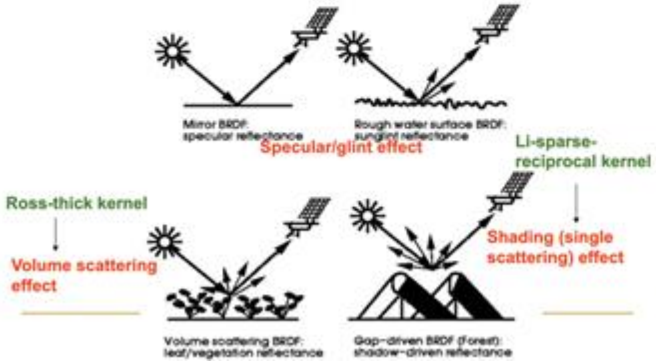
In-scattering - How to describe radiation directional properties? BRDF



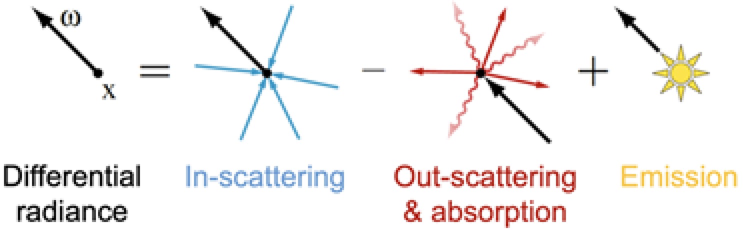
The Bi-directional Reflectance Distribution Function (BRDF) is used to describe the dependence of reflected radiation on the incident (i) and outgoing (v) directions (Nicodemus, 1977).

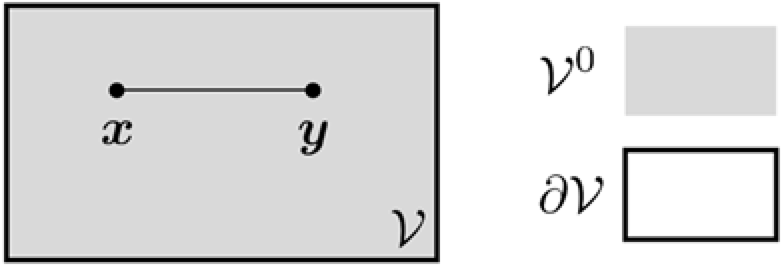
$$B(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{dL_v(\theta_i, \phi_i, \theta_v, \phi_v)}{L_i(\theta_i, \phi_i) \cos \theta_i d\Omega_i} \text{ sr}^{-1}$$





Out-scattering & absorption - How to quantify ***attenuation***? Beer's Law





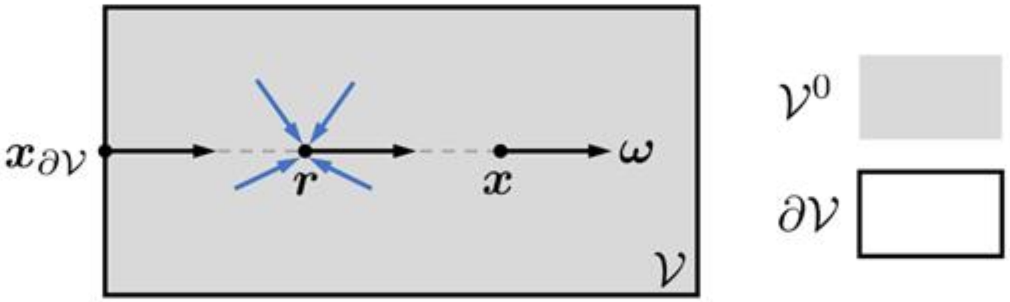
- For any $x, y \in \mathcal{V}$, the attenuation between ***x*** and ***y*** is

$$T(\boldsymbol{x} \leftrightarrow \boldsymbol{y}) := \exp\left(-\int_{(\boldsymbol{x},\boldsymbol{y})} \sigma_t(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}\right)$$

  - A line integral between ***x*** and ***y***
  - $0 \leq T(\boldsymbol{x} \leftrightarrow \boldsymbol{y}) \leq 1$ for all ***x*** and ***y***
  - For homogeneous media with $\sigma_t(\boldsymbol{x}) \equiv \sigma_t$,

$$T(\boldsymbol{x} \leftrightarrow \boldsymbol{y}) = \exp(-\|\boldsymbol{x} - \boldsymbol{y}\|\sigma_t)$$

Solving Radiative Transfer Equations - Derive Integral form of RTEs



Attenuation — In-scattering — Emission

$$L(\boldsymbol{x},\boldsymbol{\omega}) = \int_0^{h(\boldsymbol{x},\boldsymbol{\omega})} T(\boldsymbol{r}\leftrightarrow\boldsymbol{x})\left[\sigma_s(\boldsymbol{r})\int_{\mathbb{S}^2} f_p(\boldsymbol{r},\boldsymbol{\omega}_i\to\boldsymbol{\omega})L(\boldsymbol{r},\boldsymbol{\omega}_i)\,\mathrm{d}\boldsymbol{\omega}_i + Q(\boldsymbol{r},\boldsymbol{\omega})\right]\mathrm{d}\tau$$

$$+\,T(\boldsymbol{x}_{\partial\mathcal{V}}\leftrightarrow\boldsymbol{x})\,L(\boldsymbol{x}_{\partial\mathcal{V}},\boldsymbol{\omega})$$ where $\boldsymbol{r} := \boldsymbol{x} - \tau\boldsymbol{\omega}$

Attenuation — Boundary cond.

(The second term vanishes when $h(\boldsymbol{x},\boldsymbol{\omega}) = +\infty$)

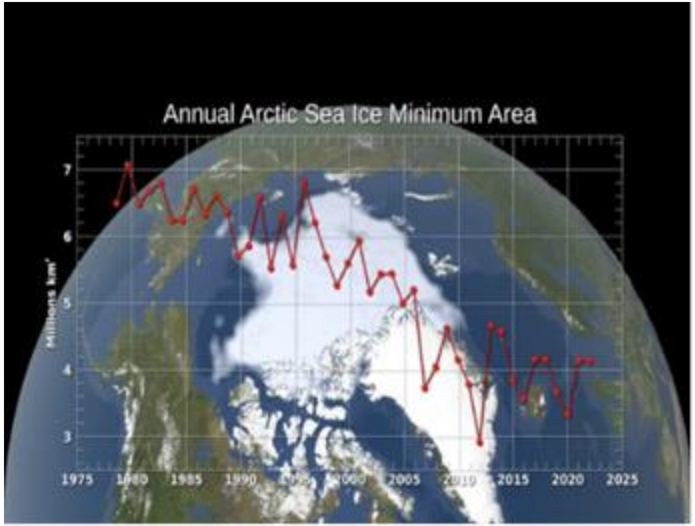
All RTMs follow this general form.

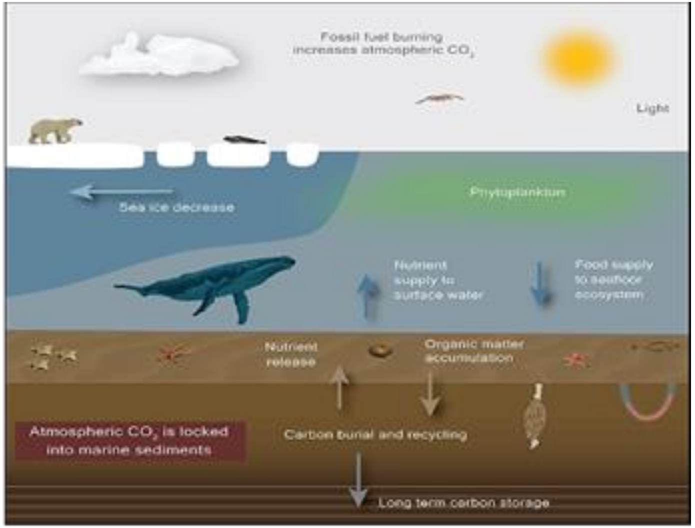
The differences however, are essentially due to the various forms for the emission and absorption coefficients.

SMRT - The Snow Microwave Radiative Transfer (SMRT)

https://smrt-model.science/documentation.html

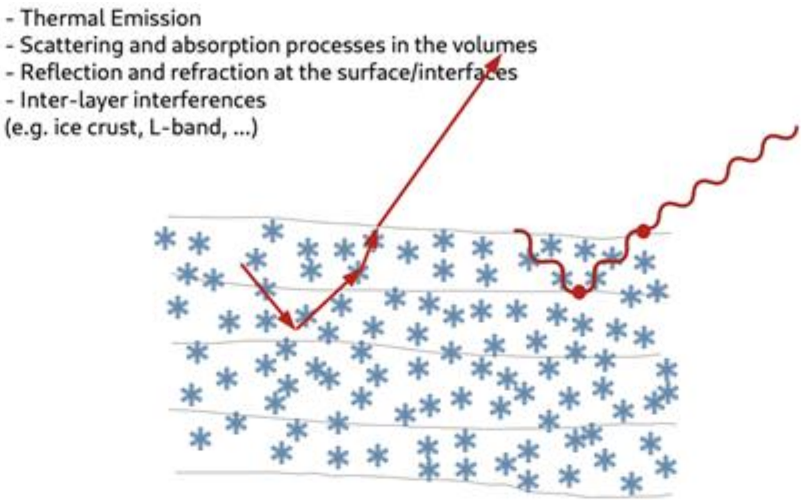
Why Northern ice/snow monitoring is important?

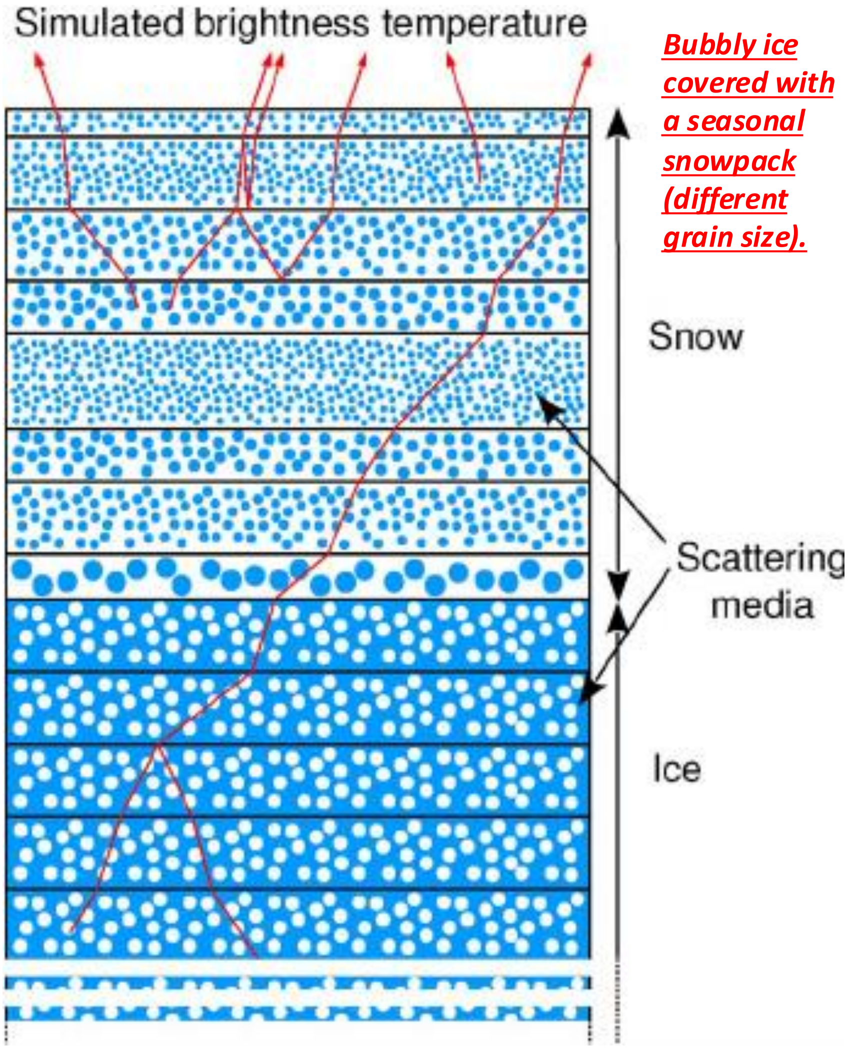




SMRT assumptions:

1. A stack of plane-parallel, horizontally infinite, homogeneous layers.
2. Layers are isotropic media at the microstructure scale as well as at the scale of the snowpack.
3. Microstructural anisotropy of snow is neglected and that structures formed by wind (sastrugi, dunes) are not taken into account yet.





Bubbly ice covered with a seasonal snowpack (different grain size).

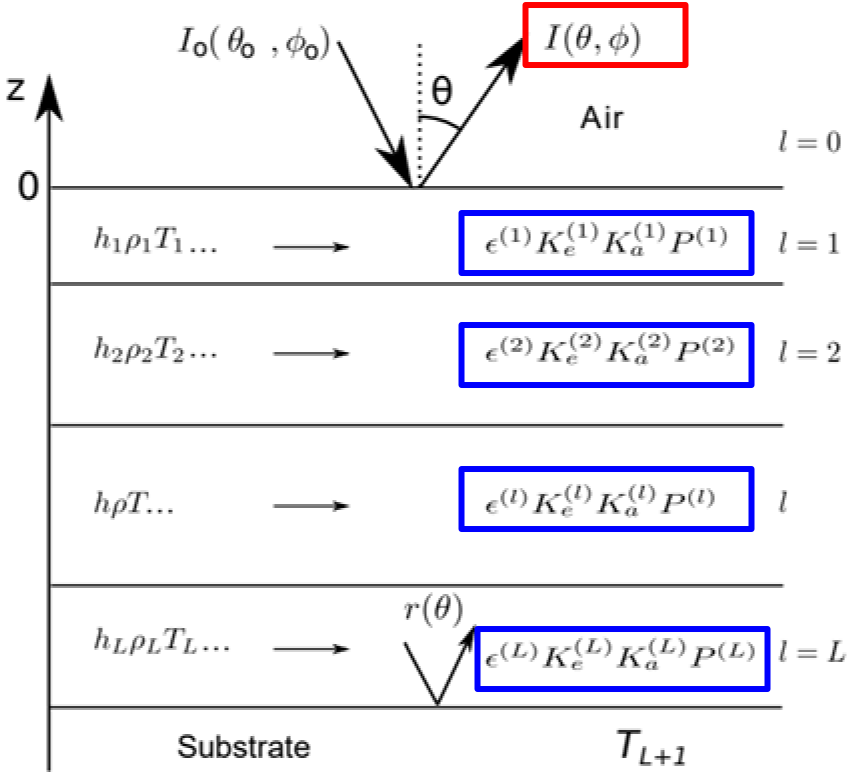
Snow Microwave Radiative Transfer (SMRT) Model

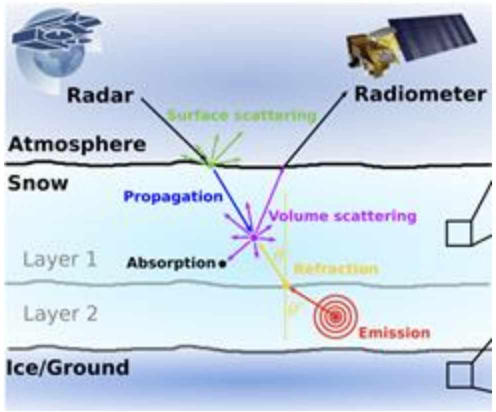
Some RTMs address the total backscattering coefficient from a multilayered snowpack (DMRT-QMS, MEMLS, etc.) in various angular and polarimetric configurations, well adapted for high-frequency radar (Ku band and higher frequencies) due to multiple scattering implementations.

SMRT addresses both backscattering and emission with targeted validity range of 1-200 GHz (not fully valid yet for the upper frequency range, and validity at Ku and Ka bands).

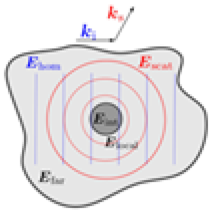
SMRT was developed to unify and inter-compare different RTMs, offering the capability of switching between different electromagnetic theories, representations of snow microstructure, and other modules involved in various calculation steps.

https://www.polarmonitoringproject.org/snow-model-radiative-transfer-smrt/





Phase function	$p = \frac{4\pi}{\kappa_s}\lvert f(\boldsymbol{k}_s, \boldsymbol{k}_i)\rvert^2$
Angular distribution of scattered intensity	
Scattering coefficient:	$\kappa_s = \frac{1}{4\pi}\int_{4\pi} d\Omega \lvert f(\boldsymbol{k}_s, \boldsymbol{k}_i)\rvert^2$
Total scattered intensity	
Extinction coefficient:	$\kappa_e = 2\mathcal{I}m(\sqrt{\varepsilon_{eff}})$
Intensity attenuation (including scattering and absorption)	
Absorption coefficient:	$\kappa_a = \kappa_e - \kappa_s,$
Intensity attenuation due to Ohmic currents	



SMRT, where h denotes the layer thickness, ρ and T denote the density and temperature of the corresponding medium, respectively, l denotes the number of layers, θ denotes the incident angle, I_0 denotes the atmospheric contribution, and I denotes the simulation result. The physical properties of snow and ice in each layer are related to the electromagnetic properties, mainly represented by permittivity (ε), absorption coefficient (Ka), extinction coefficient (Ke), and phase function (P).

SMRT - Model Input & Output?

Radiative Transfer Equations - how to describe ***the variation of the radiance L per unit distance along ω?***

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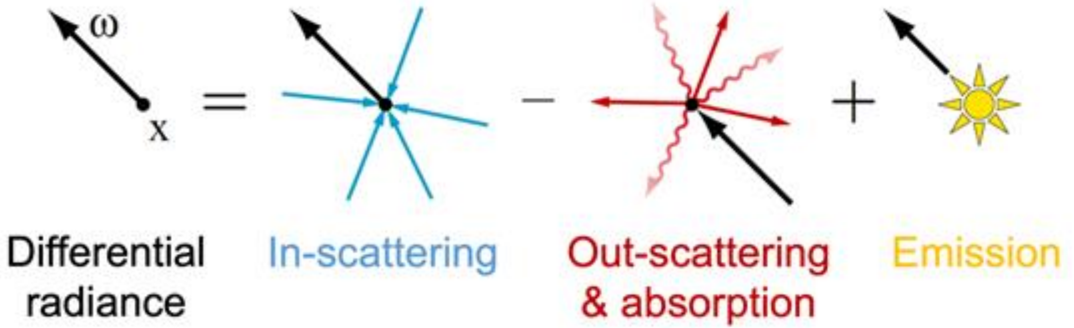

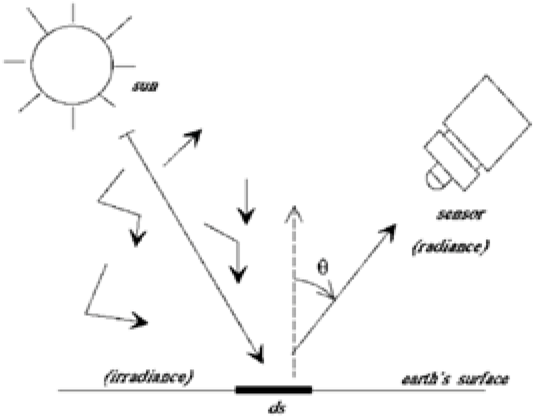

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In-scattering | Out-scattering & absorption | Emission

Scattering coefficient: $\sigma_s(\boldsymbol{x}) \in \mathbb{R}_{\geq 0}$,
Phase function: $f_p(\boldsymbol{x},\boldsymbol{\omega}_i\to\boldsymbol{\omega})$, a probability density over $\mathbb{S}^2$ given x and ω_i

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Extinction coefficient: $\sigma_t(\boldsymbol{x}) \in \mathbb{R}_{\geq \sigma_s(\boldsymbol{x})}$
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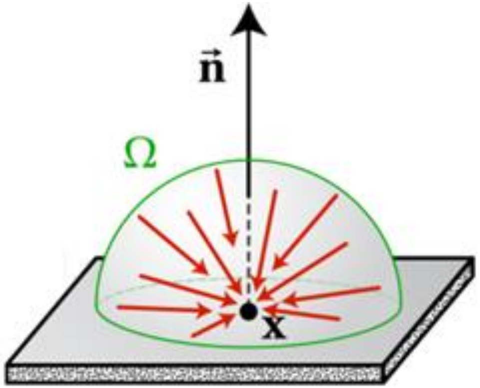
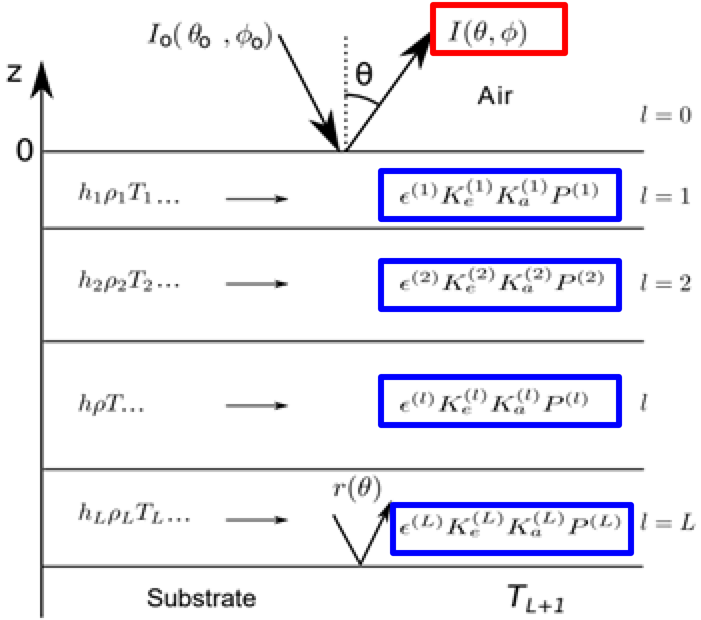
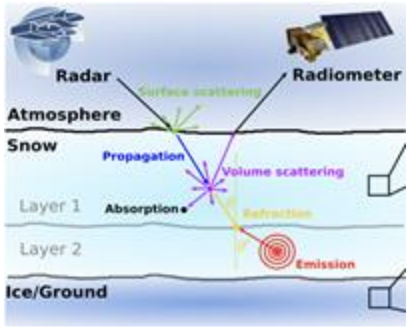
Source term: $Q(\boldsymbol{x},\boldsymbol{\omega}) \in \mathbb{R}_{\geq 0}$



- Differential radiance

$$(\boldsymbol{\omega}\cdot\nabla)L(\boldsymbol{x},\boldsymbol{\omega}) = \left.\frac{\mathrm{d}L(\boldsymbol{x}+\tau\boldsymbol{\omega},\boldsymbol{\omega})}{\mathrm{d}\tau}\right|_{\tau=0} = \lim_{\tau\to 0}\frac{L(\boldsymbol{x}+\tau\boldsymbol{\omega},\boldsymbol{\omega}) - L(\boldsymbol{x},\boldsymbol{\omega})}{\tau}$$





Phase function	$p = \frac{4\pi}{\kappa_s} \lvert f(\boldsymbol{k}_s, \boldsymbol{k}_i) \rvert^2$
	Angular distribution of scattered intensity
Scattering coefficient:	$\kappa_s = \frac{1}{4\pi} \int_{4\pi} d\Omega \lvert f(\boldsymbol{k}_s, \boldsymbol{k}_i) \rvert^2$
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	Intensity attenuation (including scattering and absorption)
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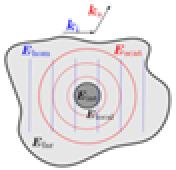
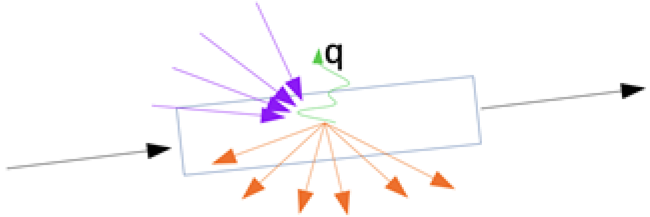
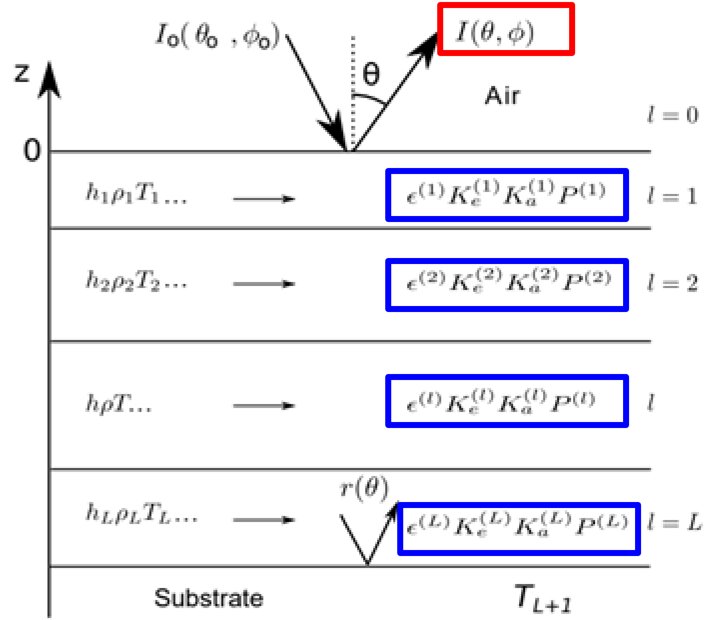
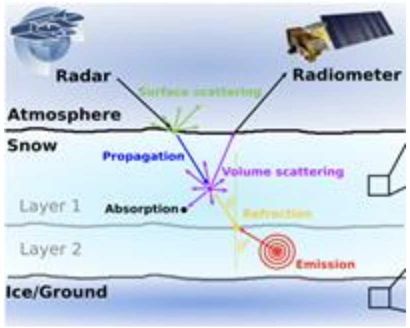


$$\mu \frac{\partial \mathbf{I}(\mu, \phi, z)}{\partial z} = -\boldsymbol{\kappa}_e(\mu, \phi, z) \mathbf{I}(\mu, \phi, z) + \frac{1}{4\pi} \iint_{4\pi} \mathbf{P}(\mu, \phi; \mu', \phi', z) \mathbf{I}(\mu', \phi', z) \, d\Omega' + \boldsymbol{\kappa}_a(\mu, \phi, z) \alpha T(z) \mathbf{1}$$



In this RTE, how to express reflection, transmission and emission?





Phase function	$p = \frac{4\pi}{\kappa_s} \lvert f(\mathbf{k}_s, \mathbf{k}_i) \rvert^2$	Angular distribution of scattered intensity
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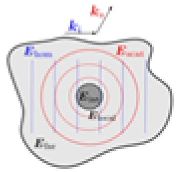
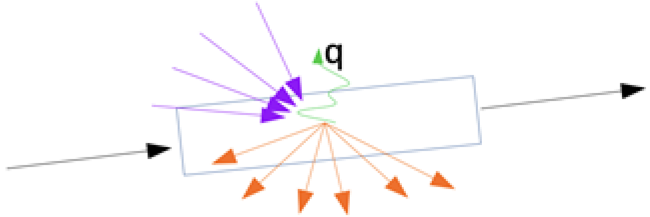


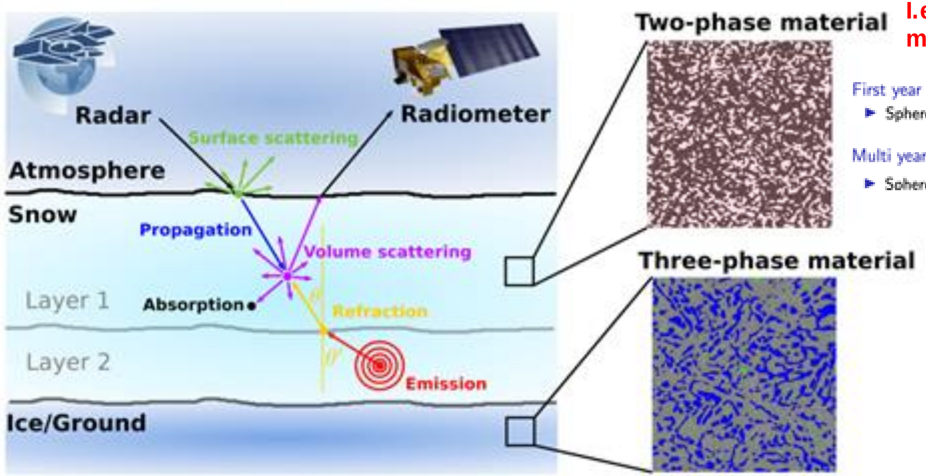
$$\mu \frac{\partial \mathbf{I}(\mu, \phi, z)}{\partial z} = -\boldsymbol{\kappa}_e(\mu, \phi, z)\mathbf{I}(\mu, \phi, z) + \frac{1}{4\pi} \iint_{4\pi} \mathbf{P}(\mu, \phi; \mu', \phi', z)\mathbf{I}(\mu', \phi', z)\, d\Omega' + \boldsymbol{\kappa}_a(\mu, \phi, z)\, \alpha T(z) \mathbf{1}$$



Solving this RTE requires knowing phase function, scattering, extinction and absorption coefficient. How?

In-scattering - How BRDF is defined by **micro-sctructure & dielectric constent** of snow and ice?

$$\mu \frac{\partial \mathbf{I}(\mu,\phi,z)}{\partial z} = -\boldsymbol{\kappa}_{\mathrm{e}}(\mu,\phi,z)\mathbf{I}(\mu,\phi,z) + \frac{1}{4\pi}\iint_{4\pi}\mathbf{P}(\mu,\phi;\mu',\phi',z)\mathbf{I}(\mu',\phi',z)\,d\Omega' + \boldsymbol{\kappa}_{\mathrm{a}}(\mu,\phi,z)\alpha T(z)\mathbf{1}$$



I.e., snow contains two materials

First year ice:
- Spheroidal brine inclusions in a pure ice background

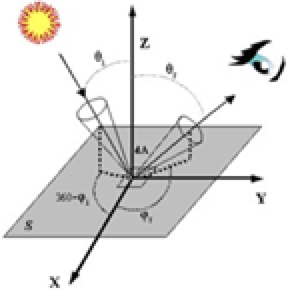
Multi year ice:
- Spheroidal air inclusions in a saline ice background

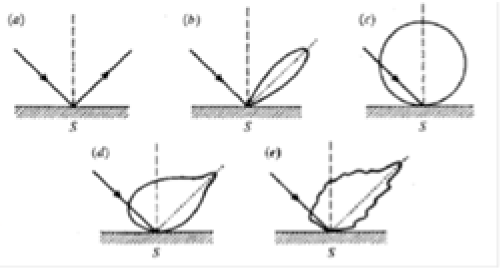
I.e., ice contains two materials (but is usually simplified to just contain two.

Scattering approximations in SMRT:
(cf. smrt.emmodel)
- QCA: Quasicrystalline approximation
- QCA-CP: Quasicrystalline approximation (coherent potential)
- SFT: Strong fluctuation theory
- IBA: Improved Born approximation
- SCE: Strong contrast expansion

The Bi-directional Reflectance Distribution Function (BRDF) is used to describe the dependence of reflected radiation on the incident (i) and outgoing (v) directions (Nicodemus, 1977).

$$B(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{dL_v(\theta_i, \phi_i, \theta_v, \phi_v)}{L_i(\theta_i, \phi_i)\cos\theta_i d\Omega_i}\ \mathrm{sr}^{-1}$$





Rayleigh phase function in the IBA:

$$f_{scat}(\chi) \sim M(|k_d|)\ k^4\ \sin^2\chi\ F_{\mathrm{IBA}}(\varepsilon_1, \varepsilon_2)$$

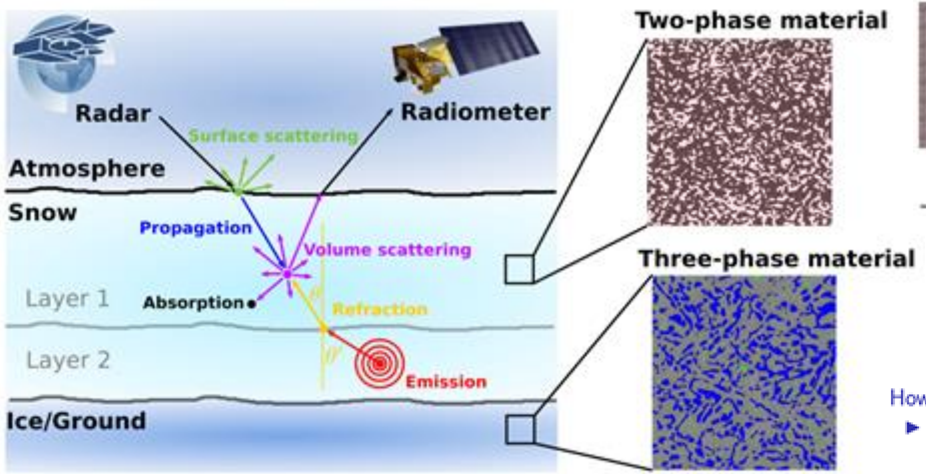
- Size term
- Angle term
- Dielectric term

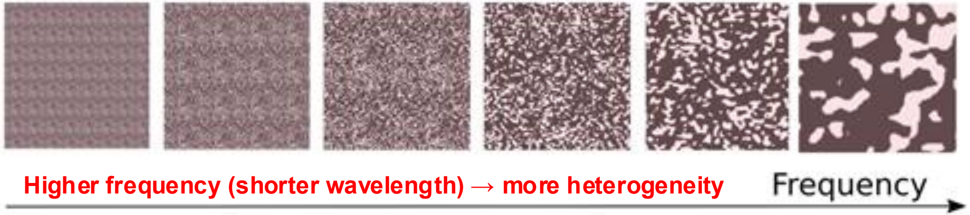
$$\mathrm{P}(\mu,\phi,\mu',\phi') = \begin{bmatrix} P_{11} & P_{12} & P_{13} & 0 \\ P_{21} & P_{22} & P_{23} & 0 \\ P_{31} & P_{32} & P_{33} & 0 \\ 0 & 0 & 0 & P_{44} \end{bmatrix}$$

can be computed from $f_{scat}(\chi)$ (details in Picard et al 2018)

Attanuation - How scattering/absorbtion is defined by **micro-sctructure & dielectric constent** of heterogeneous mixtures (i.e., air, water, brine and ice)?

$$\mu \frac{\partial \mathbf{I}(\mu,\phi,z)}{\partial z} = -\boldsymbol{\kappa}_{\mathrm{e}}(\mu,\phi,z)\mathbf{I}(\mu,\phi,z) + \frac{1}{4\pi}\iint_{4\pi} \mathbf{P}(\mu,\phi;\mu',\phi',z)\mathbf{I}(\mu',\phi',z)\,d\Omega' + \boldsymbol{\kappa}_{\mathrm{a}}(\mu,\phi,z)\alpha T(z)\mathbf{1}$$





For "very low" frequency:

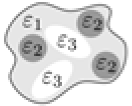
- Snow or saline ice can be regarded as a homogeneous medium described by an **effective permittivity** (rigorous concept)
- Effective permittivity contains microstructure only via volume fractions ("grain size" does not enter).

How effective permittivities are mostly derived:

- Place randomly oriented spheroids with permittivity $\varepsilon_2, \varepsilon_3$ in a background medium ε_1.

Permittivity formulations in SMRT:
(cf. `smrt.permittivity`)

- Polder–van Santen (default)
- Bruggemann
- Maxwell–Garnett
- + many others



superposes background field (hom) and scattered field (scat)

$$\boldsymbol{E} = \boldsymbol{E}_{hom} + \boldsymbol{E}_0 f(\boldsymbol{k}_s, \boldsymbol{k}_i)\frac{\exp ikr}{r}$$

Rayleigh phase function in the IBA:

$$f_{scat}(\chi) \sim M(|k_d|)\, k^4\, \sin^2\chi\, F_{\mathrm{IBA}}(\varepsilon_1, \varepsilon_2)$$

- Size term
- Angle term
- Dielectric term

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0 \exp(ik\hat{\boldsymbol{k}}\cdot\boldsymbol{r} - i\omega t)$$

with a complex **propagation constant** k

$$k = k' + ik''$$

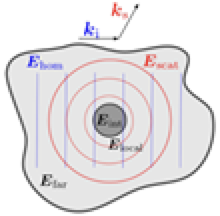
which is related to the complex **index of refraction** n

$$k = nk_0$$

which is in turn related to the complex **dielectric constant** ε (or **permittivity**)

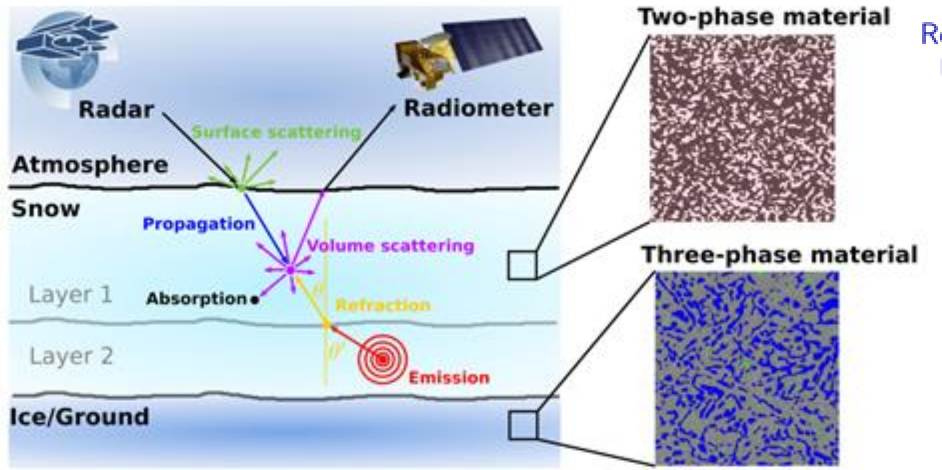
$$\varepsilon = n^2$$

All quantities k, n, ε are equivalent, complex-valued, EM material properties.



Interface/surface scattering/refraction - How are they defined by **micro-sctructure & dielectric constent**

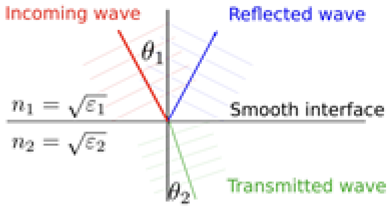
$$\mu \frac{\partial \mathbf{I}(\mu,\phi,z)}{\partial z} = -\boldsymbol{\kappa}_e(\mu,\phi,z)\mathbf{I}(\mu,\phi,z) + \frac{1}{4\pi}\iint_{4\pi} \mathbf{P}(\mu,\phi;\mu',\phi',z)\mathbf{I}(\mu',\phi',z)\,d\Omega' + \boldsymbol{\kappa}_a(\mu,\phi,z)\,\alpha T(z)\mathbf{1}$$



Refraction:

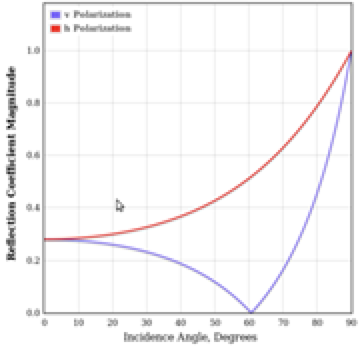
- Snells law:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

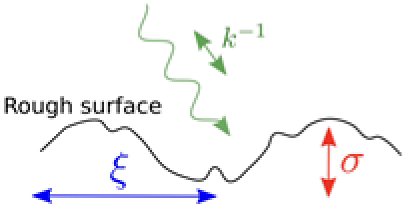


Transmission and reflection:

- Fresnel formulas
- Angles, intensities determined by the effective permittivities ε_1 ε_2 and polarization
- Whether a surface is smooth depends on k



[ULABY 2013]



Surface scattering models in SMRT

(`smrt.interface`)

- Geometrical optics $k\sigma \gg 1$, $k\xi \gg 1$, σ/ξ fixed
- Integral equation method (IEM)

- Vertical height standard deviation σ
- Horizontal correlation lenght ξ

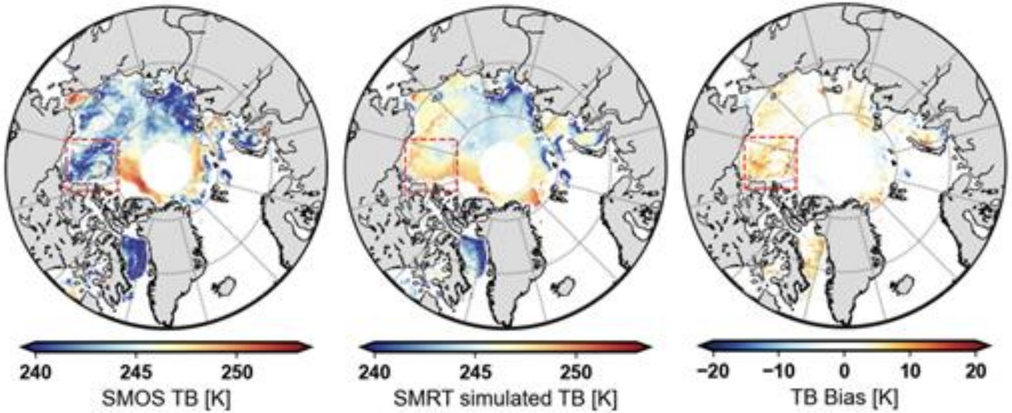
EM surface scattering theory:

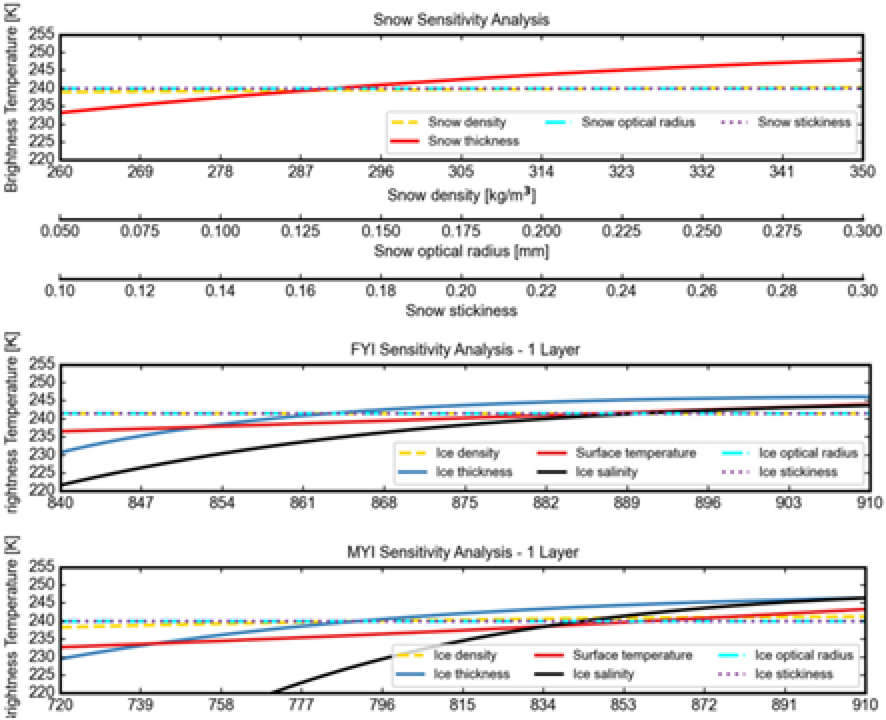
- Involves at least **two lenght scale ratios**, either ($k\sigma$, $k\xi$) or ($k\sigma$, σ/ξ)
- Approximations depend on their magnitude

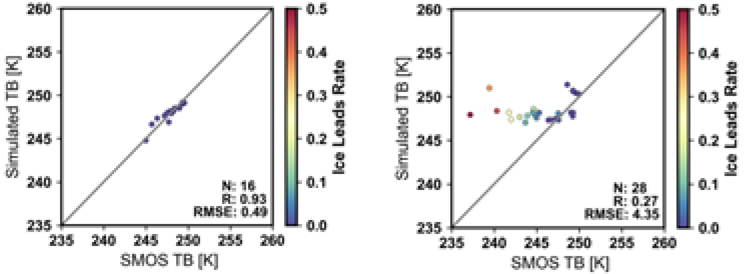
SMRT simulation (Fan et. al., 2023)

Table 1. Default values and variation ranges for sensitivity studies.

Type		Parameters	Defaults	Sensitivity Range
Sea ice	MYI	Ice thickness (m)	3	2~5
		Surface temperature (°C)	−30	−40~−25
		Ice salinity (PSU)	3.5	1~5
		Ice density (kg/m^3)	850	720~910
		Ice optical radius (mm)	0.1	0.05~0.3
		Ice stickiness	0.2	0.1~0.3
	FYI	Ice thickness (m)	1	0.5~2
		Surface temperature (°C)	−30	−40~−25
		Ice salinity (PSU)	10	5~12
		Ice density (kg/m^3)	910	840~910
		Ice optical radius (mm)	0.1	0.05~0.3
		Ice stickiness	0.2	0.1~0.3
Snow		Snow thickness (cm)	15	0~50
		Snow density (kg/m^3)	350	260~350
		Snow optical radius (mm)	0.16	0.05~0.3
		Snow stickiness	0.18	0.1~0.3







Questions?